

An Optimal Approach to Solve Rich Vehicle Routing Problem

Ismail Yusuf, Mohd. Sapiyan Baba, and Nur Iksan

Abstract—This research present a set-partitioning heuristic to generate populations in a hybrid genetic algorithm (hGA) which is able to solve four different variants of the vehicle routing problem: the multi-depot vehicle routing problem (MDVRP), the heterogeneous vehicle routing problem (HVRP), site dependent capacitated vehicle routing problem (SDCVRP) and the asymmetric vehicle routing problem (AVRP). The goal is to find a set of routes with minimum fuel consumption and satisfying a number of predetermined constraints. Initial test results show that the average of improvement rates of the general GA is lower than hGA, 9.72% vs. 19.81%. Routes produced by hGA proposed effectively used for solve rich VRP and especially vehicle routing problem with a large number of customers, depots and vehicles.

Keywords— Multi-depot, site-dependent, heterogeneous, asymmetric, VRP, clustering, heuristic, hybrid GA.

I. INTRODUCTION

THE vehicle routing problem (VRP) is a general combinatorial optimization problem that has become a key component of transportation management. The VRP was first introduced by Dantzig and Ramser [5]. The general VRP consists of determining several vehicle routes with minimum cost for serving a set of customers, whose geographical coordinates and demands are known in advance. Each customer is required to be visited only once by one vehicle. Typically, vehicles are homogeneous and have the same capacity restriction. The general VRP is defined on a connected graph G . Let $G = (V, A)$ be a graph where $V = \{v_0, v_1, v_2, \dots, v_n\}$ is a vertex set and $A = \{(v_i, v_j) \mid v_i, v_j \in V, i < j\}$ is the set of arcs. Vertices v_0 correspond to the depot at which $K = \{k_0, k_1, k_2, \dots, k_n\}$ is a set homogeneous vehicles are based, and the remaining vertices denote the customers. Since the vehicles are homogeneous, the capacity for all vehicles is equal and denoted by Q . The total size of deliveries for customers assigned to each vehicle must not exceed the

vehicle capacity (Q_i). With every arc (i, j) is associated a non-negative distance matrix $C = (c_{ij})$, which represents the travel distance from v_i to v_j . The general VRP considered to be symmetric, i.e., $c_{ij} = c_{ji}$; $i, j = 0, 1, 2, \dots, n$. The problem is to construct a minimum travel cost, feasible set of routes - one for each vehicle. A route is a sequence of locations that a vehicle must visit along with the indication of the service it provides. The vehicle must start and finish its tour at the same depot. The VRP may actually be considered a broad class of routing problem. Composed of many specific variants i.e. multi depot VRP [9, 10], capacitated VRP, site dependent [1, 2, 4], and symmetric VRP [11].

The algorithm proposed in this paper is applied to four different problems: the multi-depot vehicle routing problem (MDVRP), the heterogeneous vehicle routing problem (HVRP), site dependent capacitated vehicle routing problem (SDCVRP) and the asymmetric vehicle routing problem (AVRP).

The capacitated vehicle routing problem (CVRP) is a problem raised in the fields of transportation in which a fleet of delivery vehicles must service known customer demands for a single commodity from a common depot at minimum cost. The vehicles are homogeneous, having the same capacity and costs, while the size of the fleet is unlimited. One variant of the CVRP is the heterogeneous vehicle routing problem (HVRP) [8].

Site dependent capacitated vehicle routing problem (SDCVRP), is a variant of the heterogeneous capacitated VRP where there exists a dependency between the type of vehicle and the customer, meaning that not every type of vehicle can serve every type of customer because of site-dependent restrictions [1,2,3,4]. The general VRP uses the symmetric vehicle routing problem (SVRP). In real life this is a one way traffic problem, the path used to travel from customer i to customer j is different that used to travel from customer j to customer i . In one way traffic, we must use AVRP [7].

II. PROBLEM FORMULATION

Let, $G = (P, A)$ be a graph. Where, P is the set of all ports (customer and fuel ports). It is constituted by the nodes C (customer ports) and by the nodes D (fuel ports) at which K is a set mix vehicles with capacity q_k are based. $A = \{(i, j) \mid i, j \in P, i < j\}$ is the set of arcs. Every arc (i, j) is associated with a non-negative distance matrix $L = (l_{ij})$, which represents the asymmetric travel distance from i to j , i.e., $l_{ij} \neq l_{ji}$; $i, j \in C \cup D$. The problem is to construct route with minimum fuel

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consumption; feasible set of routes for each vehicle.

In order to present the mathematical formulation of the models, we use the following:

$C = \{1, 2, 3, \dots, c\}$ is the set of customer ports.

$D = \{c+1, c+2, c+3, \dots, d\}$ is the set of fuel ports.

$P = \{1, 2, 3, \dots, c, c+1, c+2, c+3, \dots, d\}$; $C \cup D$ is the set of all ports; indexed p . Where i used to indexed departure port and j used indexed arrive port.

$K = \{1, 2, 3, \dots, k\}$ is the set of the vehicles, indexed by k .

$R = \{1, 2, 3, \dots, r\}$ is the set of routes, indexed by r .

r is the feasible route for ship k to serve ports without exceeding the constraints:

1. Travel time of the trip for any vehicle is not longer than the maximum allowed routing time T
2. Travel distance of the trip for any vehicle is not longer than the maximum allowed routing distance.
3. Passenger on board should not exceed capacity of the ship k .
4. The feasible route r has a minimum of one fuel port d .

The mathematical formulation is given below:

$$\text{minimize } z = \sum_{k \in K} \sum_{r \in R_k} f_r^k \cdot u_r^k + \sum_{i, j \in P} \sum_{k \in K} \sum_{r \in R_k} \alpha \cdot y_{ij}^{kr} \cdot u_r^k \quad (1)$$

Subject to;

$$\sum_{k \in K} w_i^{kr} \cdot u_r^k \geq 1, \quad \forall i \in P, \forall k \in K, \forall r \in R_k \quad (2)$$

$$\sum_{r \in R} u_r^k = 1, \quad \forall k \in K, \forall r_k \in R_k \quad (3)$$

$$\sum_{r \in R} t_r^k \leq T, \quad \forall k \in K \quad (4)$$

$$\sum_{r \in R_k} l_r^k \leq L, \quad \forall k \in K \quad (5)$$

$$t_r^k = \frac{l_r^k}{V_k}, \quad \forall k \in K, \forall r \in R_k \quad (6)$$

$$\sum_{i=1} q_i^k \cdot w_i^{kr} \leq q_k, \quad \forall i \in P, \forall k \in K, \forall r \in R_k \quad (7)$$

$$\sum_{i, j, p \in P} \sum_{k \in K} y_{ij}^{kr} \geq 0.75 \quad (8)$$

$$\sum_{k \in K} \sum_{r \in R} d_r^k w_i^{kr} \geq 1 \quad (9)$$

$$u_r^k, w_i^{kr}, y_{ij}^{kr} \in \{0, 1\}, \quad \forall (i, j) \in P, \forall k \in K, \forall r \in R_k \quad (10)$$

The objective function minimizes the sum of the fuel consumption travelled, the penalty cost associated with constraint violation respect to the penalty cost associated with minimum load factor (α). Equation (2) ensures that all ports (customer and fuel port) i are serviced by ship k in the established route r minimum at once; equation (3) ensures that one ship k is assigned for serving a route r ; equation (4) ensures that travel time of the route r for ship k is not longer than the maximum allowed routing time T , $T = 14$ days;

equation (5) ensures that travel distance of the route r for ship k is not longer than the maximum allowed routing distance L ; equation (6) represents time from i to j equals to the distance from i to j is divided by running speed V_k ; equation (7) is the vehicle capacity constraint; equation (8) ensure that load factor of route is equal or more than 0.75 and the other restriction is (9) ensure that route r served by ships k should possess a fuel-port. Constraint (10) imposes binary requirements on the variables.

III. PROBLEM SOLUTION

Problem solving in this study include into two steps, first step is how to generate a feasible route combinations that meets all the requirements, and the second step is how the route combination in the first step has an optimal value.

The first step adopted CAB-method was proposed by Ismail *et al* [12]. CAB-method procedure involving three algorithm phases are clustering, assigned vehicle and find the best routes by combination of feasible solution. To obtain the optimal combination of routes used GA.

As VRP is an NP-hard problem, it is not easily solved. For this reason a stochastic-based search methods have been proposed. By using stochastic-based search techniques, an optimal solution can be easily found for a large-sized problem in a reasonable amount of time. Among stochastic search techniques, one of them called genetic algorithms (GA) has already been applied to solve different VRPs successfully. The continuing improvement in the performance value of GA has made them attractive for many types of problem solving optimization methods. The Hybrid-GA proposes following Fig. 1.

In order to demonstrate the effectiveness of GA, Lau *et al* [6] was comparing GA with the other various search techniques; branch and bound, simulated annealing and tabu search. They use 33 benchmark problems were selected for testing and examine its search performance. Three different scenarios i.e. 5 depots and 50 customers, 15 depots and 150 customers, 25 depots and 250 customers, were produced for the simulations. From the research by Lau *et al* [6] founded that the search performance of standard GA was generally better than that of branch and bound, simulated annealing, and tabu search in all of three scenarios.

In this study, the general GA was modified to maximize performance of GA as show in Fig.1. The modifications are included:

- Initial population is not generated randomly but based on partitioning heuristic.
- Selection process was done based on rank and selection rate.
- Uses a method that ensures current chromosomes are no worse than the previous chromosomes, it is called improvement procedure.

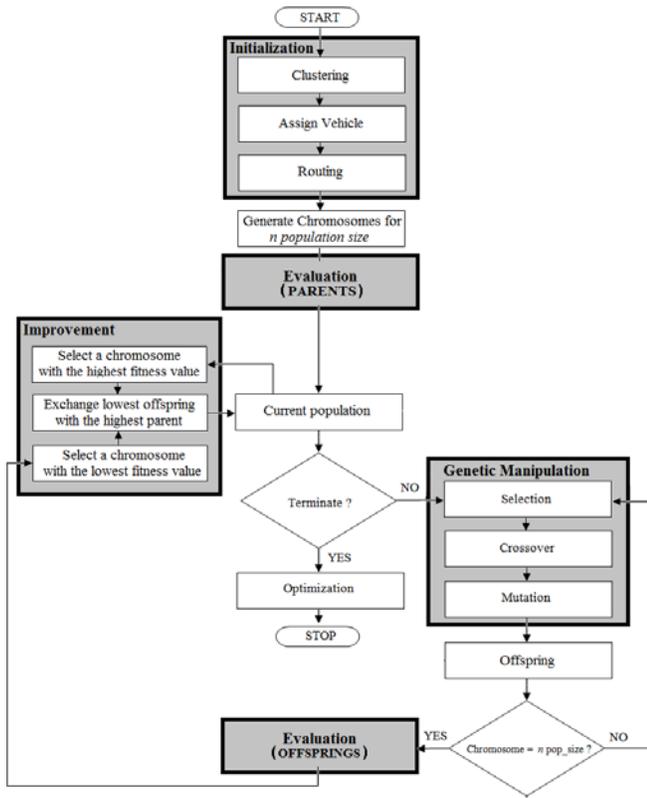


Fig. 1 hGA Proposed

A. Gene Representation

How to represent chromosomes is a key issue in the AG, in which the chromosomes could be a binary or real number. This study uses real numbers for representing the chromosomes. The use of real numbers does not require an encoding and decoding process. It was providing a solution directly and it could save the computer memory and time-saving operation. The chromosomes represented as show in Fig. 2.

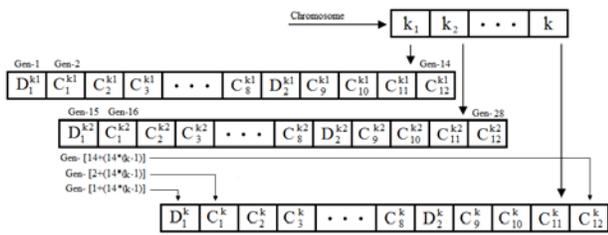


Fig. 2 Chromosome Representation

B. Initial Population Method

In this study, initial population was generated base on partitioning heuristic as described in CAB-method [12].

C. Evaluation

Evaluation is a process of calculating the objective function for each chromosome. It intended to calculate value of the fitness of each individual after genetic manipulation process. The fitness calculation is a measure to know what the best particular solution to resolve the problem is. The fitness function is the basic survival of the fittest premise among GA.

An individual was evaluated, based on a certain function as the measurement performance. In this research, chromosomes were evaluated base on fuel consumption. Objective function to minimize fuel consumption calculated by:

$$F_k^r = \eta * (P * nM) * \frac{l_r}{V_k} * \mu \tag{11}$$

where,

- F_k^r Fuel consumption of ship k when serve route r
- η High Speed Diesel constant (0.16)
- P Engine power (HP)
- nM Number of engine
- l_r Sailing distance of route r
- V_k Speed of the ship (knots)
- μ Efficiency (0.8)

Then the total fuel consumption of the whole routes is the sum of the fuel consumption all route. The fitness function:

$$F = \frac{1}{\sum F_k^r + 1} \tag{12}$$

D. Selection

Selection is a process to select parent chromosomes and offspring based on the fitness value to form a new better generation follow the objective function. In this paper, a selection method proposed to solve routing problems, namely rank and selected base on selection rate. The selection proposed procedure is as follows:

- Step 1: Generate a random number r in the range (0, 1].
- Step 2: If $r \leq P_s$ then chromosome X_h is selected.
- Step 3: Check for the number of chromosome not selected
- Step 4: Choose the chromosome with the best fitness in the previous population till current population size is p_size .

E. Crossover

Crossover is a genetic operation which is adopted to exchange information between two chromosomes for genetic exploration. Not all chromosomes are chosen for performing crossover. Whether a chromosome is selected or not is determined by a crossover rate p_m .

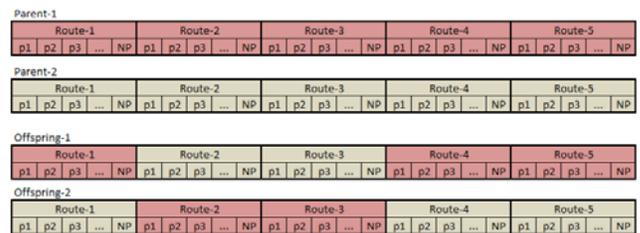


Fig. 3 Multi point crossover method

In this paper, multi point crossover was used [1]. Multi point crossover obtained using the two points in which chosen randomly. First, randomly selecting two crossover points; then exchanging the gene segment between two points of the parent gene; finally, using the gene codes on the corresponding location of parent gene to replace gene in the

other parent. The steps for the multi-point crossover process are as follows:

- Step 1: Select the crossover by using the crossover probability from the selected two parent chromosomes.
- Step 2: Select the crossing point(s) by using the crossover probability and generate two children.
- Step 3: The 1st part and 3rd which were not selected from parent 2 inherit to child 1 are exchanged and the 2nd part which were not selected from parent 1 inherit to child 2 are exchanged.

F. Mutation

Mutation is another genetic operation which provides diversity for the solutions so as to prevent them from falling into local optima. Not all genes are chosen for performing mutation. Mutation process occurs for a living creature can survive with better quality. In the GA, mutation process is to obtain a better chromosome to remain in the selection process and hopefully it will be perilously close to the optimum solution. Instead the process of gene mutations that obtain a worse chromosome could make the chromosomes were eliminated in the selection process. Whether a gene is selected or not is determined by a mutation rate p_m . In this paper, shift neighbour was proposed. Shift neighbour is shift randomly genes code in a chromosome to a neighbour routes (i.e. neighbour with respect to the numbering for the routes).

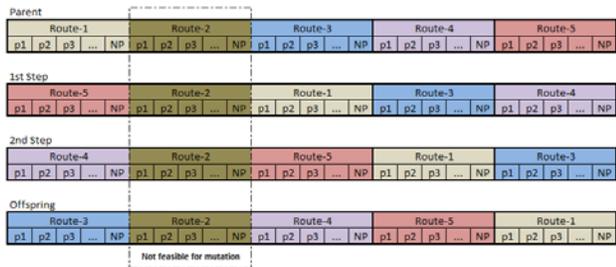


Fig. 4 Shift neighbour mutation method

The steps for the shift neighbour mutation process in this study are as follows:

- Step 1: Select the genes in a route which will be generated mutation by random.
- Step 2: Change the genes in a route with the next routes.
- Step 3: Genes in the 1st route are exchanged with the 2nd route, and the 2nd route group are exchanged with the 3rd group.

IV. EXPERIMENTS

A. Experiment Design

In order to show the effectiveness of hGA proposed, simulations were carried out. In this study, the 11 benchmark tested. All the benchmark shows in Table I.

The algorithm proposes coded in java and, using a Intel(R) Core(TM) i5 CPU M430 @ 2.27GHz. As methods compared have a stochastic behaviour, they have been tested 50 times on each benchmark for every GA operator. The best found values over 50 runs were kept and the average of the best values was

calculated.

TABLE I
BENCHMARK

Benchmark	Number of			Fuel Consumption (Liters)
	Customer Port	Fuel Port	Vehicle	
40c-9d-8k	40	9	8	1,275,883
28c-9d-9k	28	9	9	2,375,323
45c-11d-11k	45	11	11	3,868,567
32c-4d-8k	32	4	8	1,036,758
34c-11d-11k	34	11	11	2,743,105
63c-14d-11k	63	14	11	4,755,085
18c-6d-8k	18	6	8	1,491,149
28c-6d-11k	28	6	11	2,134,324
12c-4d-8k	12	4	8	1,263,833
53c-12d-11k	53	12	11	2,945,322
24c-5d-10k	24	5	10	1,267,387

B. Result and Analysis

To check the quality of solution obtained over algorithms in 11 benchmarks then check efficiency was done. All result recorded in Table II.

TABLE II
FUEL CONSUMPTION OF THE BEST KNOW RESULT AND OTHER ALGORITHM

Benchmark	Fuel Consumption			
	Best Known Solution	Heuristic	general GA	hGA
40c-9d-8k	1,275,883	1,122,712	1,044,496	954,654
28c-9d-9k	2,375,323	2,064,836	1,860,811	1,711,743
45c-11d-11k	3,868,567	3,340,013	2,967,359	2,680,247
32c-4d-8k	1,036,758	919,118	869,112	798,467
34c-11d-11k	2,743,105	2,377,556	2,132,196	1,930,129
63c-14d-11k	4,755,085	4,095,004	3,624,502	3,269,042
18c-6d-8k	1,491,149	1,308,901	1,205,546	1,121,831
28c-6d-11k	2,134,324	1,858,045	1,681,237	1,526,019
12c-4d-8k	1,263,833	1,114,330	1,037,452	994,332
53c-12d-11k	2,945,322	2,549,070	2,281,022	2,063,132
24c-5d-10k	1,267,387	1,116,445	1,039,177	950,480

In this section, a computational study is carried out to compare the general GA and hGA. The performance of the general GA and hGA is evaluated using the same selection, crossover and mutation method i.e. selection method based on selection rate ($P_s = 0.5$), multi point crossover and shift neighbourhood mutation. The GA parameters for the problems are population size: 100, generation: 1000, crossover rate: 0.7, and mutation rate: 0.5, in each of which there are 11 benchmarks.

Other experiment was done to compare performance of general GA and hGA. Result summarized in Table III.

TABLE III
PERFORMANCE OF GENERAL GA AND HGA OVER 11 BENCHMARKS

Benchmark	general GA			hGA		
	Best Population in the First Generation	Best Population at the End Generation	Improvement rate (%)	Best Population in the First Generation	Best Population at the End Generation	Improvement rate (%)
40c-9d-8k	0.8907	0.9574	7.49	0.8907	1.0475	17.60
28c-9d-9k	0.4843	0.5374	10.96	0.4843	0.5842	20.63
45c-11d-11k	0.2994	0.3370	12.56	0.2994	0.3731	24.62
32c-4d-8k	1.0880	1.1506	5.75	1.0880	1.2524	15.11
34c-11d-11k	0.4206	0.4690	11.51	0.4206	0.5181	23.18
63c-14d-11k	0.2442	0.2759	12.98	0.2442	0.3059	25.27
18c-6d-8k	0.7640	0.8295	8.57	0.7640	0.8914	16.68
28c-6d-11k	0.5382	0.5948	10.52	0.5382	0.6553	21.76
12c-4d-8k	0.8974	0.9639	7.41	0.8974	1.0057	12.07
53c-12d-11k	0.3923	0.4384	11.75	0.3923	0.4847	23.55
24c-5d-10k	0.8957	0.9623	7.44	0.8957	1.0521	17.46

According to Table III, note that the average of improvement rates for the general GA is about 9.72%. The highest improvement rate is for the benchmark 45c-11d-11k while the smallest improvement rate in general GA is for the benchmark 12c-4d-8k with 12 customer 4 depots and 8 vehicles; about 5.75 %. For this study, both general GA and hGA used in partitioning set heuristic for the initial population. And since the improvement rate for general GA is

small then it could be said, the solutions generated using these partitioning heuristics set are already near optimal. So, there is only a small gap for improving the best population of the end generation in general GA. Based on the above observation, it is summarized that set partitioning heuristics could help to solve the problem of the combination of MDVRP, HFMVRP, and AVR in small benchmarks and in case GA is not used.

In both algorithms, there are no differences for the best initial solution in the first generation. This is due to the fact that the initial population is generated by the same method i.e. partitioning set heuristic for generating chromosomes. The result then changed after general GA and hGA runs.

Table III shows at the end of the generation has a difference in the final best solution. hGA can generate a much better final solution than general GA. The average of improvement rates the general GA is lower than those of hGA, 9.72% vs. 19.81%. Furthermore, the improvement rates in hGA with 11 vehicles improve markedly when compared with the benchmark with fewer vehicles. The highest improvement rate is for the benchmark which serves 63 customers, 14 depots and 11 vehicles, about 25.27%. It is about 12.29% higher than the improvement rate in the general GA for the same benchmark. The smallest improvement rate in hGA is for the benchmark which serves 12 customers, 4 depots and 8 vehicles, about 12.08%. It is about 4.67 % higher than the improvement rate in the general GA for the same benchmark.

It was found that the performance of hGA is superior to that of general GA in terms of the solution quality. This is due to the fact that hGA added with improvement procedure. In other words, the improvement procedure for the hGA plays an important role in all GA processes. Therefore, it is suggested that hGA should be adopted to solve the combination of MDVRP, HVRP, SDCVRP and AVR.

V. CONCLUSION

In this paper, the MDVRP, HVRP, SDCVRP and AVR were studied and it is called rich VRP. Problem solving in this study include into two steps, first step is how to generate a feasible route combinations that meets all the requirements, and the second step is how the route combination in the first step has an optimal value. In the first step, partitioning heuristic was used while hGA used in the second step.

In order to validate the suggested mathematical programming model and hybrid GA, it was applied to 11 benchmarks over two section experiments. Result shows that the average of improvement rates the general GA is lower than hGA, 9.72% vs. 19.81%.

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Networks, Cognitive

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